Lecture 15  Trace Element Modeling

Friday, March 11th, 2005

Trace elements are useful tools for modeling magmatic processes

- Batch Melting
- Incremental Melting
- Crystal Fractionation
- Magma Mixing
Models of Magma Evolution

- Batch Melting
  - The melt remains resident until at some point it is released and moves upward from the residue
  - Equilibrium melting process with variable % melting

\[ \frac{C_L}{C_O} = \frac{1}{D_i(1 - F) + F} \]

\( C_L \) = trace element concentration in the liquid (melt)
\( C_O \) = trace element concentration in the original rock before melting began
\( F \) = wt fraction of melt produced = melt/(melt + rock)
Batch Melting

A plot of $C_L/C_O$ vs. $F$ for various values of $D_i$

- $D_i = 1.0$

Figure 9-2. Variation in the relative concentration of a trace element in a liquid vs. source rock as a function of $D$ and the fraction melted, using equation (9-5) for equilibrium batch melting. From Winter (2001) An Introduction to Igneous and Metamorphic Petrology. Prentice Hall.

$D_i \gg 1.0$ (compatible element)

- Very low concentration in melt
- Especially for low % melting (low $F$)

Figure 9-2. Variation in the relative concentration of a trace element in a liquid vs. source rock as a function of $D$ and the fraction melted, using equation (9-5) for equilibrium batch melting. From Winter (2001) An Introduction to Igneous and Metamorphic Petrology. Prentice Hall.
Highly incompatible elements ($D_i << 1$)

- Greatly concentrated in the initial small fraction of melt produced by partial melting

- Subsequently diluted as $F$ increases

Figure 9-2. Variation in the relative concentration of a trace element in a liquid vs. source rock as a function of $D$ and the fraction melted, using equation (9-5) for equilibrium batch melting. From Winter (2001) An Introduction to Igneous and Metamorphic Petrology. Prentice Hall.

As $F \rightarrow 1$ the concentration of every trace element in the liquid = the source rock ($C_L/C_O \rightarrow 1$)

$$\frac{C_L}{C_O} = \frac{1}{D_i(1 - F) + F} \quad \text{As } F \rightarrow 1$$

C_L/C_O \rightarrow 1

Figure 9-2. Variation in the relative concentration of a trace element in a liquid vs. source rock as a function of $D$ and the fraction melted, using equation (9-5) for equilibrium batch melting. From Winter (2001) An Introduction to Igneous and Metamorphic Petrology. Prentice Hall.
As \( F \to 0 \), \( \frac{C_L}{C_O} \to \frac{1}{D_i} \)

If we know \( C_L \) of a magma derived by a small degree of batch melting, and we know \( D_i \), we can estimate the concentration of that element in the source region (\( C_O \)).

\[
\frac{C_L}{C_O} = \frac{1}{D_i(1 - F) + F}
\]

For a basalt (~1% melt) with Ba = 700 ppm, \( D_{Ba} \approx 0.01 \) for mantle, then:

\[
C_O = C_L / (1/0.01) = 700 / 100 = 7
\]

Therefore mantle source contains about 7 ppm Ba.

- For very incompatible elements as \( D_i \to 0 \),

\[
\frac{C_L}{C_O} = \frac{1}{D_i(1 - F) + F}
\]

simplifies to

\[
\frac{C_L}{C_O} = \frac{1}{F}
\]

If we know the concentration of a very incompatible element in both a magma and the source rock, we can determine the fraction of partial melt produced.

If a tholeiitic basalt with ~90 ppm Ba is produced by melting of mantle with 7 ppm Ba, then:

\[
F = \frac{C_O}{C_L} = \frac{7}{90} = 0.078
\]

Therefore the tholeiite was produced by about 8% melting.
Worked Example of Batch Melting: Rb and Sr
Basalt with the mode:

Table 9-2  Conversion from mode to weight percent

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Mode</th>
<th>Density</th>
<th>Wt prop</th>
<th>Wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td>ol</td>
<td>15</td>
<td>3.6</td>
<td>54</td>
<td>0.18</td>
</tr>
<tr>
<td>cpx</td>
<td>33</td>
<td>3.4</td>
<td>112.2</td>
<td>0.37</td>
</tr>
<tr>
<td>plag</td>
<td>51</td>
<td>2.7</td>
<td>137.7</td>
<td>0.45</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>303.9</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

1. Convert to weight % minerals (\(W_{\text{ol}}\) \(W_{\text{cpx}}\) etc.)

2. Use equation \(D_i = \sum W_i \overline{D}_i\)

and the table of \(D\) values for Rb and Sr in each mineral to calculate the bulk distribution coefficients:
Compatibility depends on minerals and melts involved.

Which are incompatible? Why?

Table 9-1. Partition Coefficients (C_i/C_L) for Some Commonly Used Trace Elements in Basaltic and Andesitic Rocks

<table>
<thead>
<tr>
<th></th>
<th>Olivine</th>
<th>Opx</th>
<th>Cpx</th>
<th>Garnet</th>
<th>Plag</th>
<th>Amph</th>
<th>Magnetite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rb</td>
<td>0.010</td>
<td>0.022</td>
<td>0.031</td>
<td>0.042</td>
<td>0.071</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Sr</td>
<td>0.014</td>
<td>0.040</td>
<td>0.060</td>
<td>0.012</td>
<td>1.830</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>Ba</td>
<td>0.010</td>
<td>0.013</td>
<td>0.026</td>
<td>0.023</td>
<td>0.23</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>14</td>
<td>5</td>
<td>7</td>
<td>0.955</td>
<td>0.01</td>
<td>6.8</td>
<td>29</td>
</tr>
<tr>
<td>Cr</td>
<td>0.70</td>
<td>10</td>
<td>34</td>
<td>1.345</td>
<td>0.01</td>
<td>2.00</td>
<td>7.4</td>
</tr>
<tr>
<td>La</td>
<td>0.007</td>
<td>0.03</td>
<td>0.056</td>
<td>0.001</td>
<td>0.148</td>
<td>0.544</td>
<td>2</td>
</tr>
<tr>
<td>Ce</td>
<td>0.006</td>
<td>0.02</td>
<td>0.092</td>
<td>0.007</td>
<td>0.082</td>
<td>0.843</td>
<td>2</td>
</tr>
<tr>
<td>Nd</td>
<td>0.006</td>
<td>0.03</td>
<td>0.230</td>
<td>0.026</td>
<td>0.055</td>
<td>1.340</td>
<td>2</td>
</tr>
<tr>
<td>Sm</td>
<td>0.007</td>
<td>0.05</td>
<td>0.445</td>
<td>0.102</td>
<td>0.039</td>
<td>1.804</td>
<td>1</td>
</tr>
<tr>
<td>Eu</td>
<td>0.007</td>
<td>0.05</td>
<td>0.474</td>
<td>0.243</td>
<td>0.1/1.5*</td>
<td>1.557</td>
<td>1</td>
</tr>
<tr>
<td>Dy</td>
<td>0.013</td>
<td>0.15</td>
<td>0.582</td>
<td>1.940</td>
<td>0.023</td>
<td>2.024</td>
<td>1</td>
</tr>
<tr>
<td>Er</td>
<td>0.026</td>
<td>0.23</td>
<td>0.583</td>
<td>4.700</td>
<td>0.020</td>
<td>1.740</td>
<td>1.5</td>
</tr>
<tr>
<td>Yb</td>
<td>0.049</td>
<td>0.34</td>
<td>0.542</td>
<td>6.167</td>
<td>0.023</td>
<td>1.642</td>
<td>1.4</td>
</tr>
<tr>
<td>Lu</td>
<td>0.045</td>
<td>0.42</td>
<td>0.506</td>
<td>6.950</td>
<td>0.019</td>
<td>1.563</td>
<td></td>
</tr>
</tbody>
</table>

Data from Rollinson (1993). * Eu^3+/Eu^2+ italics are estimated

Worked example

\[
D_{Rb} = (Wt_{Oli} \times D_{Rb}) + (Wt_{Cpx} \times D_{Rb}) + (Wt_{Plag} \times D_{Rb})
\]

\[
D_{Rb} = (0.18 \times 0.010) + (0.37 \times 0.031) + (0.45 \times 0.071)
\]

\[
D_{Rb} = 0.045
\]

\[
D_{Sr} = (Wt_{Oli} \times D_{Sr}) + (Wt_{Cpx} \times D_{Sr}) + (Wt_{Plag} \times D_{Sr})
\]

\[
D_{Sr} = (0.18 \times 0.014) + (0.37 \times 0.060) + (0.45 \times 1.830)
\]

\[
D_{Sr} = 0.848
\]
3. Use the batch melting equation

\[ \frac{C_L}{C_O} = \frac{1}{D_L(1-F) + F} \]

to calculate \( \frac{C_L}{C_O} \) for various values of \( F \)

<table>
<thead>
<tr>
<th>( F )</th>
<th>( D_{Rb} )</th>
<th>( D_{Sr} )</th>
<th>( Rb/Sr )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>9.35</td>
<td>1.14</td>
<td>8.19</td>
</tr>
<tr>
<td>0.1</td>
<td>6.49</td>
<td>1.13</td>
<td>5.73</td>
</tr>
<tr>
<td>0.15</td>
<td>4.98</td>
<td>1.12</td>
<td>4.43</td>
</tr>
<tr>
<td>0.2</td>
<td>4.03</td>
<td>1.12</td>
<td>3.61</td>
</tr>
<tr>
<td>0.3</td>
<td>2.92</td>
<td>1.10</td>
<td>2.66</td>
</tr>
<tr>
<td>0.4</td>
<td>2.29</td>
<td>1.08</td>
<td>2.11</td>
</tr>
<tr>
<td>0.5</td>
<td>1.89</td>
<td>1.07</td>
<td>1.76</td>
</tr>
<tr>
<td>0.6</td>
<td>1.60</td>
<td>1.05</td>
<td>1.52</td>
</tr>
<tr>
<td>0.7</td>
<td>1.39</td>
<td>1.04</td>
<td>1.34</td>
</tr>
<tr>
<td>0.8</td>
<td>1.23</td>
<td>1.03</td>
<td>1.20</td>
</tr>
<tr>
<td>0.9</td>
<td>1.10</td>
<td>1.01</td>
<td>1.09</td>
</tr>
</tbody>
</table>


4. Plot \( \frac{C_L}{C_O} \) vs. \( F \) for each element

Figure 9-3. Change in the concentration of Rb and Sr in the melt derived by progressive batch melting of a basaltic rock consisting of plagioclase, augite, and olivine. From Winter (2001) An Introduction to Igneous and Metamorphic Petrology. Prentice Hall.
Incremental Batch Melting

- Calculate batch melting for successive batches (same equation)
- Must recalculate $D_i$ as solids change as minerals are selectively melted (computer)

Fractional Crystallization

1. Crystals remain in equilibrium with each melt increment.
2. The equation is exactly the same as for batch melting.

$$\frac{C_L}{C_o} = \frac{1}{D_i(1 - F) + F}$$
Rayleigh fractionation
The other extreme: separation of each crystal as it formed = perfectly continuous fractional crystallization in a magma chamber

Concentration of some element in the residual liquid, $C_L$, is modeled by the Rayleigh equation:

$$\frac{C_L}{C_O} = F^{(D^{-1})}$$

Rayleigh Fractionation

$C_O$ is the initial concentration in the parental melt
$F$ is the fraction of melt remaining
Crystal Fractionation

Note how compatible elements ($D > 1$) are rapidly depleted in the melt with crystallization. On the other hand incompatible ($D < 1$) elements increase gradually initially but much more rapidly as the melt is exhausted.

$C_L/C_O = F^{(D - 1)}$

For highly incompatible element ($D \sim 0.01$) the equation approximates to:-

$C_L/C_O = 1/F$ or $F = C_O/C_L$

This is a very handy tool! We can quickly estimate the extent of crystallization between two related magmas.

If the parental magma contains 0.4% $K_2O$ and the evolved magma contains 0.6% $K_2O$, then:-

$F = 0.4/0.6 = 0.67$

The extent of crystallization will be $(1 - F) = (1 - 0.67) = 0.33$
Other models are used to analyze
- Mixing of magmas
- Wall-rock assimilation
- Zone refining
- Combinations of processes