General Comments:

I. Note on significant figures: The actual rule is “when multiplying, dividing, or taking roots, the result should have the same number of significant figures as the least precise number in the calculation” (ref: http://www.wellesley.edu/Chemistry/stats/sigfig2.html)

In #1 a-d, the number of significant figures in the least precise number is 2 (K).

When using significant figures is not specified (at least in the case of this class), a judgment call is usually in order. Don’t report the answer to 10 significant figures, but more than 1 is usually necessary!

A common example of this was #4a -- it's really not practical here to claim that you know how many cubic feet are going to be removed down to the foot (would you really argue that 14,126 ft³ are going to be removed, but there is no chance that 14,127 ft³ might be removed?) but at the same time you want to be more precise than 1.0 x 10⁴ ft³. A practical medium might be 1.41 x 10⁴ ft³.

No points were taken off for significant figures in problems other than #1 in this homework, just something to keep in mind in the future.

1) 15 points

First calculate the conductivity using the permeability, density, gravity, and viscosity. Assume that the clay is essentially impermeable.

\[ K = 0.11 \text{ cm/s} = 0.0011 \text{ m/s}; \]
\[ dh = 20 \text{ m}, dl = 2000 \text{ m}; \]
\[ dh/dl = 0.01; \]
\[ A = 8 \text{ m} \times 1000 \text{ m} = 8000 \text{ m}^2; \]
\[ Q = 0.0011 \text{ m/s} \times 8000 \text{ m}^2 \times 0.01 = 0.088 \text{ m}^3/\text{s} \]

The travel time that it would take for a solute is calculated using the average linear velocity and the distance from the trench to the stream.

\[ Q/A = 0.088 \text{ m}^3/\text{s} / 8000 \text{ m}^2 = q = 0.000011 \text{ m/s} \]
\[ v = q/n = 0.000011 \text{ m/s} / 0.20 = 0.000055 \text{ m/s} \]
\[ \text{time} = \frac{\text{distance}}{v} = \frac{2000 \text{ m}}{0.000055 \text{ m/s}} = 1.15 \text{ yrs} \]

2) 25 points

Given: \( V_T = 50.00 \text{ cm}^3 \)
\[ m_{\text{orig}} = 95.80 \text{ g} \]
\[ m_{\text{dry}} = 90.25 \text{ g} \]
\[ m_{\text{sat}} = 100.20 \text{ g} \]

a) Calculate porosity (10 points)

\[ n = V_v / V_T \]

When the sample is saturated, assume the void spaces are completely filled with water.

Therefore, \( V_v = V_w \)

The mass of the water in the pores is \([m_{\text{sat}} - m_{\text{dry}}] = 100.20 - 90.25 \text{ g} = 9.95 \text{ g} \)

Find \( V_w \) by dividing the density of water.

\[ 9.95 \text{ g} / [1.0 \text{ g/cm}^3] = 9.95 \text{ cm}^3 \]

\[ n = V_v / V_T = 9.95 \text{ cm}^3 / 50.00 \text{ cm}^3 = 0.199 \]

b) Calculate the saturation ratio as it was in the field (5 points)

\[ R_s = V_w / V_v \]

Find the mass of the water in the soil when it was in the field.

\[ m_w = m_{\text{orig}} - m_{\text{dry}} = 95.80 - 90.25 \text{ g} = 5.55 \text{ g} \]

Convert this mass to \( V_w \).
\[ v_w = \frac{m_w}{\rho_w} = \frac{(5.55 \text{ g})}{(1.0 \text{ g/cm}^3)} = 5.55 \text{cm}^3 \]
\[ R_v = \frac{v_w}{v_v} = \frac{(5.55 \text{ cm}^3)}{(9.95 \text{ cm}^3)} = 0.558 \text{ or } 55.8\% \]

c) \( \rho_{\text{tce}} = 1.63 \text{ g/cm}^3 \); soak dry sample with TCE \((10 \text{ points})\)

The volume of TCE is equal to that of voids when the soil is saturated with TCE.
\[ v_{\text{tce}} = v_v = 9.95 \text{ cm}^3 \]

** A different way to find this volume is by using the calculated porosity
\[ v_{\text{tce}} = 0.199 \times 50.00 \text{ cm}^3 = 9.95 \text{ cm}^3 \] **

The mass of TCE in the saturated soil will be
\[ m_{\text{tce}} = v_{\text{tce}} \times \rho_{\text{tce}} = 9.95 \text{ cm}^3 \times 1.63 \text{ g/cm}^3 = 16.2 \text{ g} \]

The total mass after the dry sample is saturated with TCE is
\[ m_{\text{tot}} = m_{\text{tce}} + m_{\text{drysoil}} = 16.2 \text{ g} + 90.25 \text{ g} = 106.45 \text{ g} = 106.5 \text{ g} \]

3) \(30 \text{ point}\)

\[ S = 8 \times 10^{-4}; n = 0.25, \beta = 4.4 \times 10^{-10} \text{ m}^2/\text{N}, b = 200 \text{ m} \]

\((1)\) \( A = 1 \times 10^6 \text{ m}^2, \Delta h = 1 \text{ m} \) \((10 \text{ points})\)

Using the definition of storage coefficient, the volume of water is found by:
\[ V = S \times (A \times \Delta h) = (8 \times 10^{-4}) \times (10^6 \text{ m}^2) \times (1 \text{ m}) = 800 \text{ m}^3 \]
\[ (800 \text{ m}^3) \times (35.31 \text{ ft}^3/\text{m}^3) = 2.8 \times 10^4 \text{ ft}^3 \]

b) \((10 \text{ points})\)

\[ S = S_b; S_v = \rho g(\alpha + n\beta) \]

If the matrix is incompressible, then \( \alpha = 0 \), and we can use the \( S_v \) definition equation to solve for the aquifer’s specific storage, then multiply by aquifer thickness to get \( S \)

Assuming \( g = 9.8 \text{ m/s}^2; \rho = 1000 \text{ kg/m}^3; \) using \( \alpha = 0 \) and the values for \( n \) and \( \beta \) given in the problem,

\[ \rho = 9.8 \times 10^{-3} \text{ m}^2/\text{N} \]
\[ \rho = 1.1 \times 10^{-6} \text{ kg/s}^2 \cdot \text{N} \]
\[ S = S_b \times b = (1.1 \times 10^{-6} \text{ m}^{-1}) \times (200 \text{ m}) = 2.2 \times 10^{-4} \]

\((2)(10 \text{ points})\)

\[ S = S_b, \text{ so } S = S/b = 8 \times 10^{-4}/200 \text{ m} = 4 \times 10^{-6} \text{ m}^{-1} \]

\[ \alpha = \frac{S_v}{\rho g} - n\beta = 4 \times 10^{-6} \frac{1000 \times 9.8}{10^{-6}} - 0.25 \times 4.4 \times 10^{-10} = 3.0 \times 10^{-10} \text{ m}^2/\text{N} \]

4) \(20 \text{ points}\)

a) i) The aquifer skeleton expands. By assuming that \( \alpha \) is constant, we assume that changes in fractional volume (volume per unit total volume) are related linearly to changes in net (effective) stress (the stress not taken up by the water). As the stress decreases, aquifer skeleton volume will increase proportionally (but with an opposite sign).

b) (i) Water itself expands. By assuming that \( \beta \) is constant, we assume that changes in the fractional volume of water are linearly related to changes in water pressure. As the water pressure decreases, water volume increases proportionally.

c) (iii), (iv) Aquifer grain expansion and aquifer consolidation are ignored in the equation defining specific storage. We assume that the individual mineral grains are incompressible, so they will not expand. We assume that the aquifer behaves elastically that repeated loading and unloading (recharging and pumping) will not reduce the pore space.
d) We assume that response (iii) can be ignored. By making this assumption, water will be the only thing responding to a reduction in pressure within the aquifer. If we did not assume this, then a change in volume when pressure was reduced would be due partly to grain expansion and partly to water expansion.

5) 10 points
Given: \( b = 73 \text{ m} \)
\( \alpha = 10^{-8} \text{m}^2/\text{N} \)
\( \beta = 4.4 \times 10^{-10} \text{m}^2/\text{N} \)
\( n = 0.13 \)
\( h_{\text{init}} = 100 \text{ m} \)
Known: \( \rho_{\text{water}} = 1000 \text{ kg/m}^3 \)
\( g = 9.81 \text{ m/s}^2 \)

initial hydraulic head 100m

confining layer 73 m

bottom of the aquifer (datum)

You will start to dewater the aquifer when the head level reaches the clay (confining) layer, or when \( h = 73 \text{ m} \). Therefore, \( \Delta h = h_{\text{init}} - h_{\text{final}} = 100 - 73 = 27 \text{ m} \).

Equations:
\[ S_s = \rho_{\text{water}} \cdot g \cdot (\alpha + n \beta) \]
\[ S = b \cdot S_s \]
By definition:
\[ S = V \text{ drained per (unit) area per (unit) head drop} \]
or \( S = V / (A \cdot \Delta h) \)

\[ V = S \cdot A \cdot \Delta h \]
Substitute \( S = Ss \cdot b \):
\[ V = Ss \cdot b \cdot A \cdot \Delta h \]
Substitute \( Ss = \rho_{\text{water}} \cdot g \cdot (\alpha + n \beta) \)
\[ V = \rho_{\text{water}} \cdot g \cdot (\alpha + n \beta) \cdot b \cdot A \cdot \Delta h \]

We want to know \( V/A \)
\[ V / A = \rho_{\text{water}} \cdot g \cdot (\alpha + n \beta) \cdot b \cdot \Delta h \]
\[ V / A = (1000 \text{ kg/m}^3) \cdot (9.81 \text{ m/s}^2) \cdot (10^{-8} + .13 \cdot (4.4 \times 10^{-10})) \cdot (73 \text{ m}) \cdot (27 \text{ m}) \]

\[ V / A = 0.19 \text{ m} \]

Water is from expansion of the water and expansion of the aquifer skeleton.