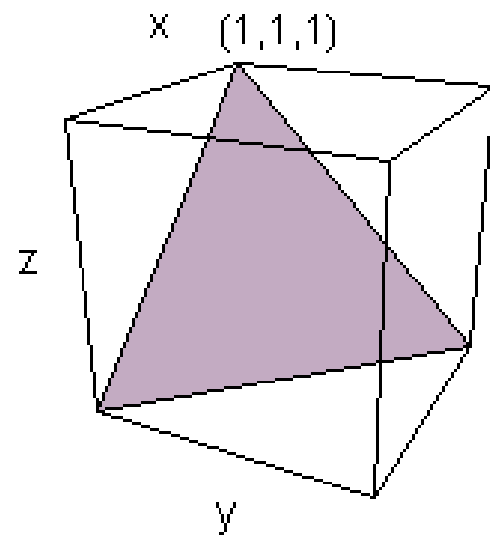
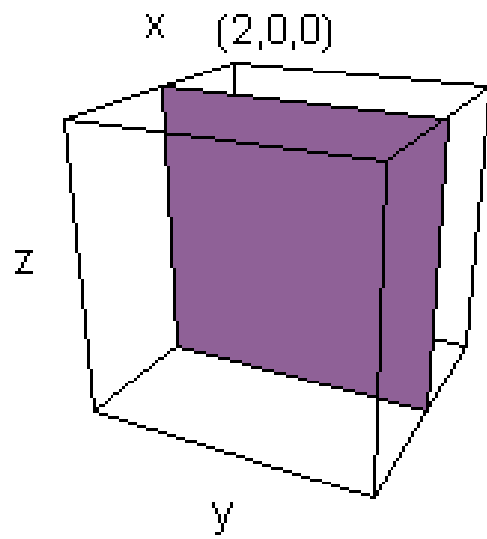
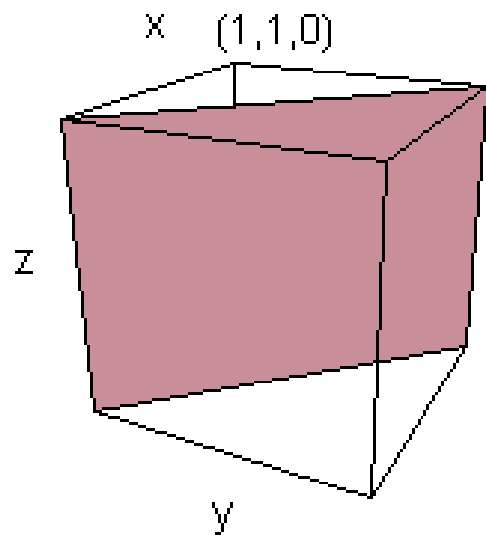
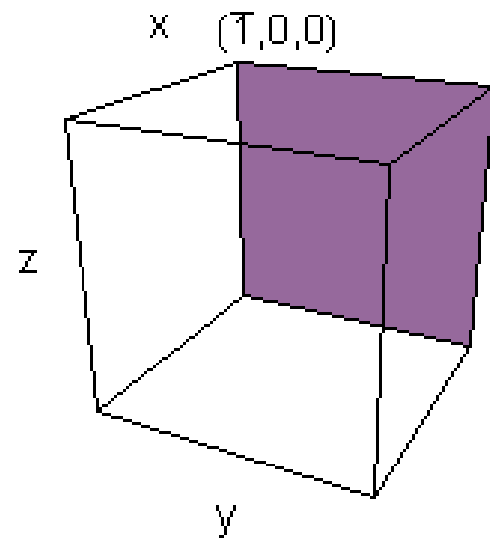
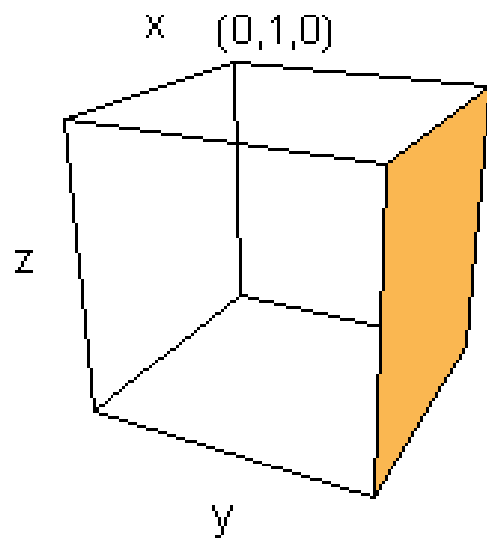
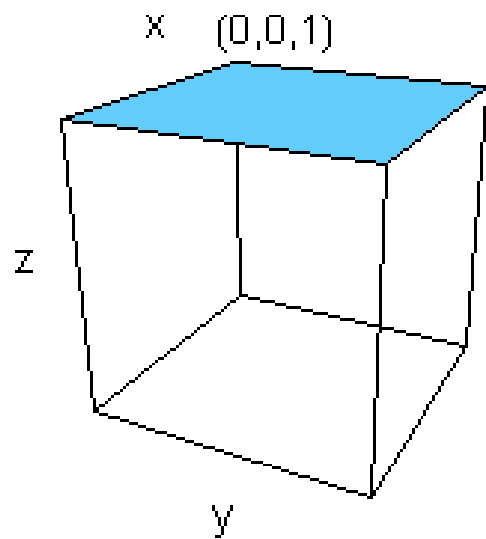
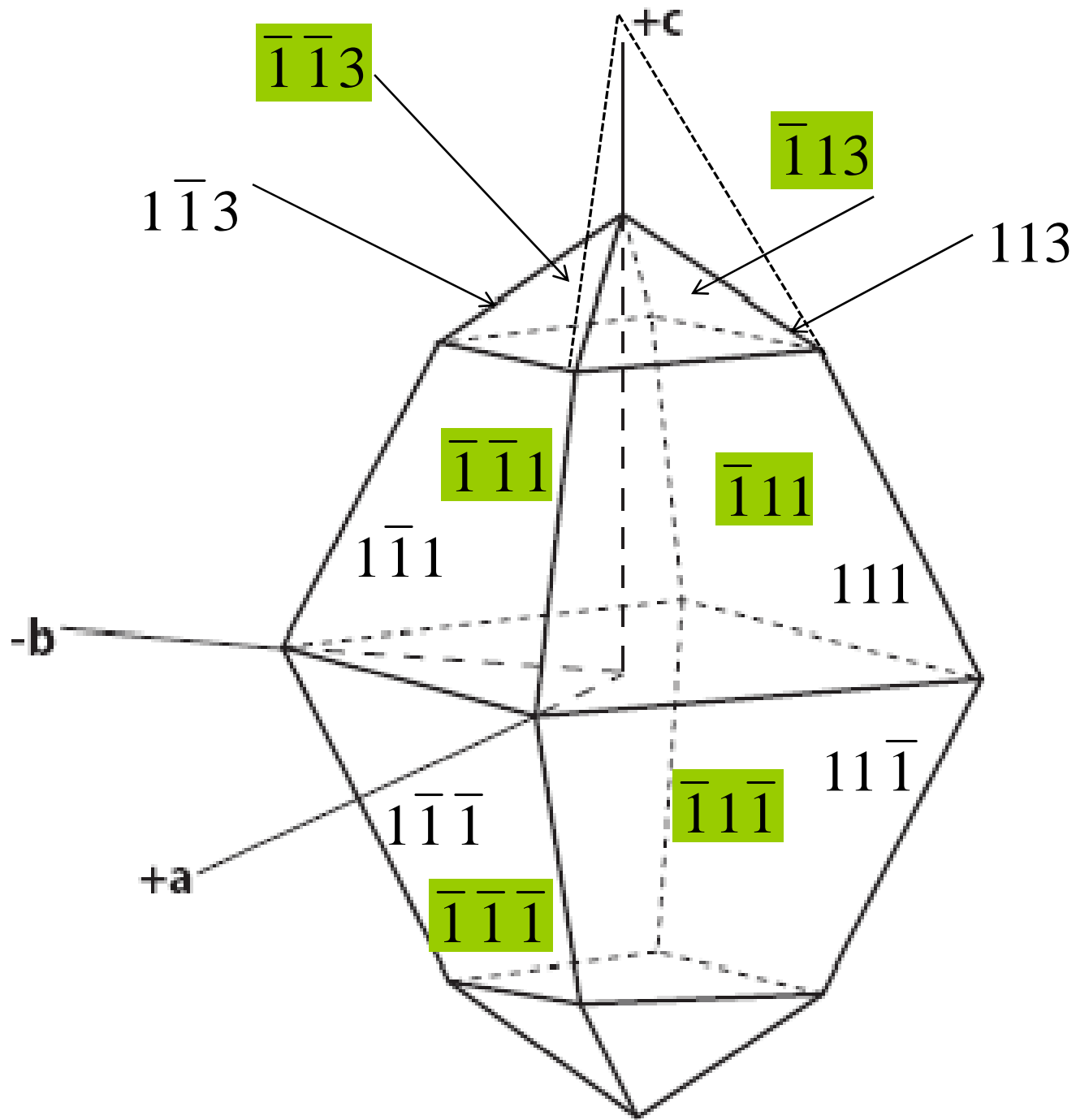


# Mathematical Symmetry

Symmetry operations  
and Indices

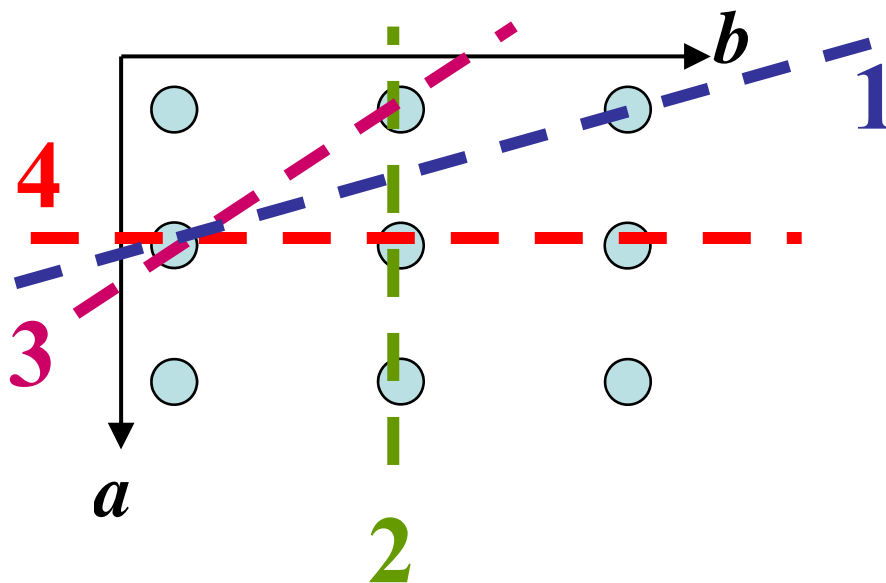




# Face Intercepts

Each face of a crystal may be defined as a function of its intercept with one or more of the crystallographic axes.

Working in the orthorhombic system looking down the c-axis

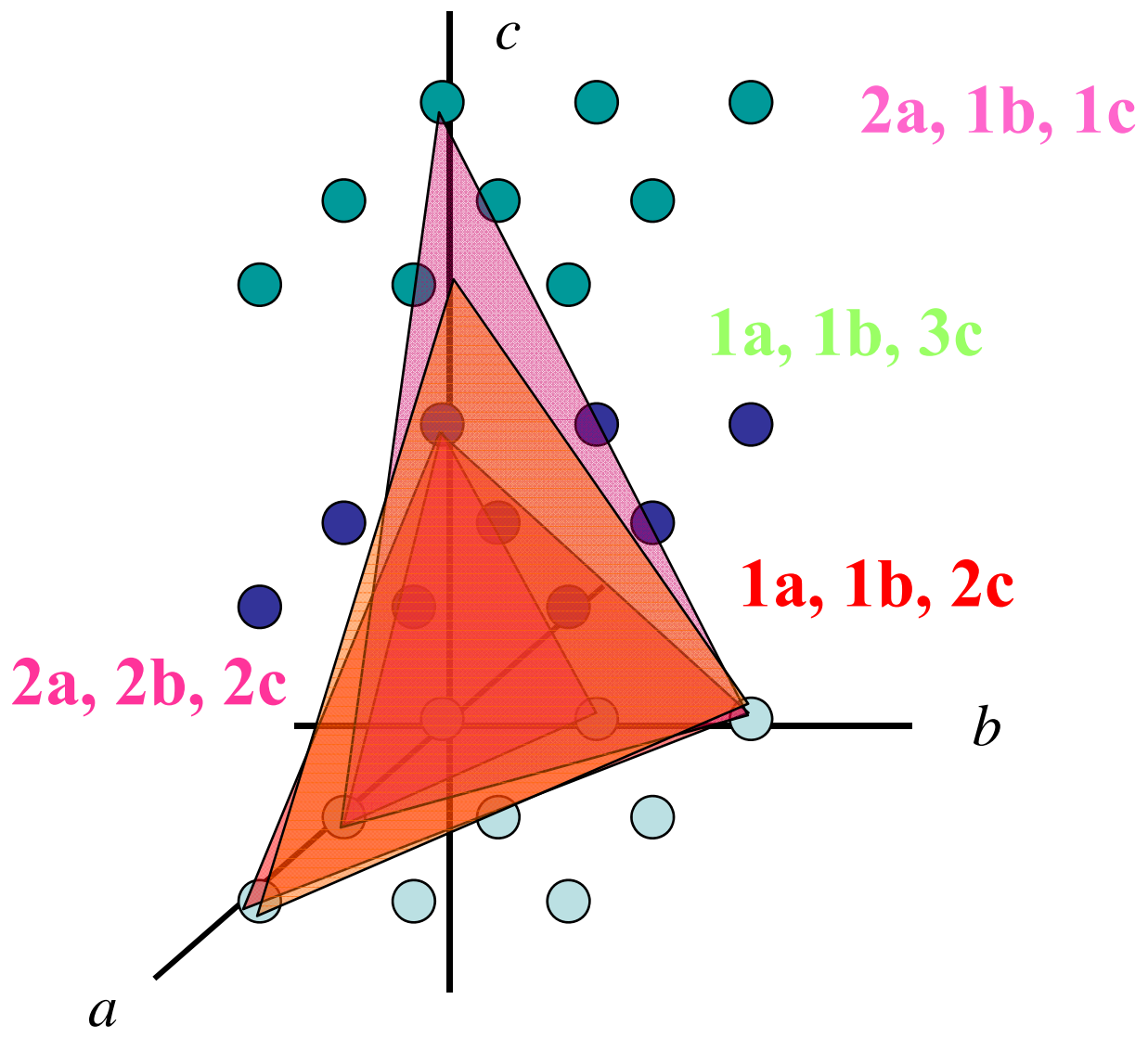


1.  $1a, 1b, \infty c$

2.  $\infty a, 1b, \infty c$

3.  $1a, 2b, \infty c$

4.  $1a, \infty b, \infty c$

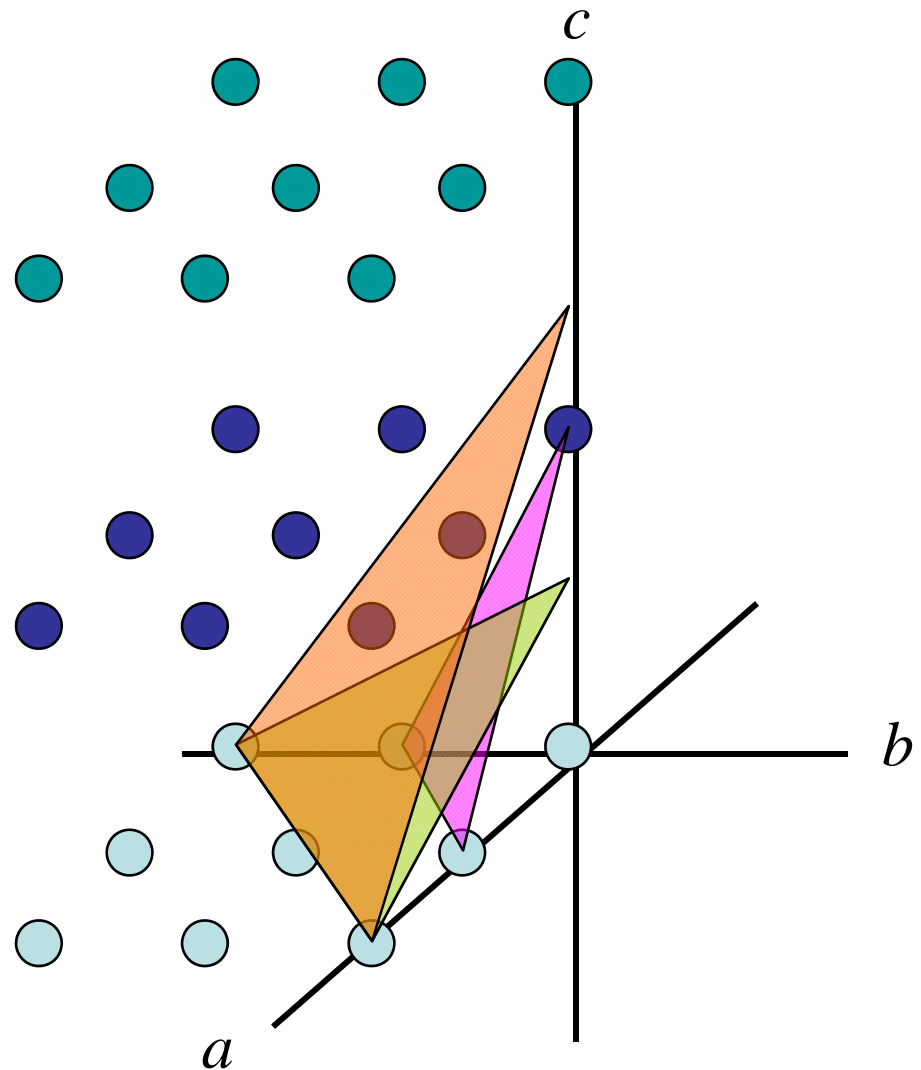


The universally employed system for notation of crystal face intercepts was proposed by W.H. Miller.

“Miller indices of a face consist of a series of whole numbers that have been derived from the intercepts of their inversions, and if necessary, the clearing of fractions”.

Each of these Miller indices intersect the  $-ve$   $b$  axis.

They are similarly defined as elements of symmetry – with bars.



# Crystal Symmetry

The external shape of a crystal reflects the presence or absence of translation-free symmetry elements in its unit cell.

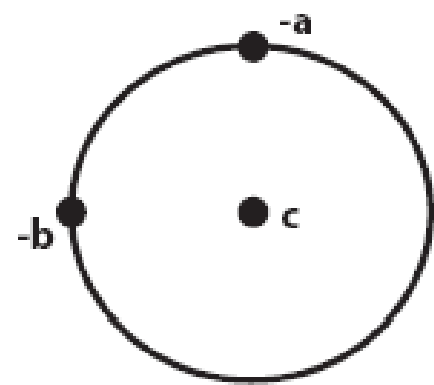
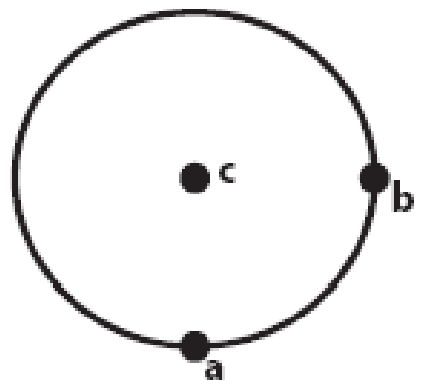
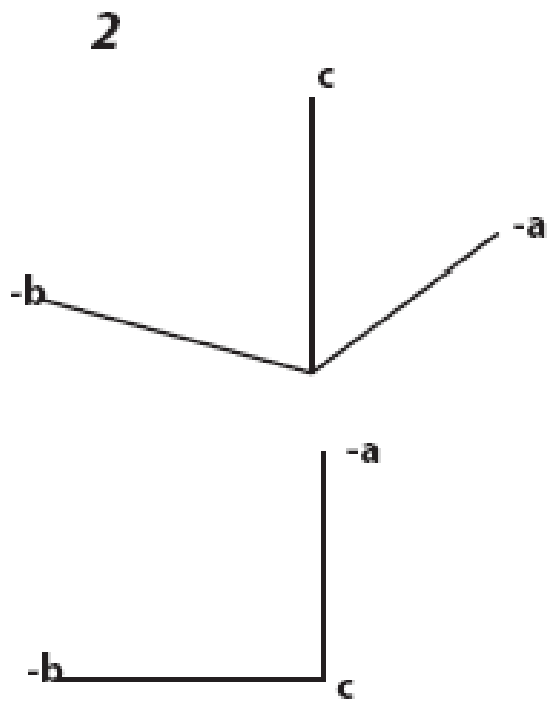
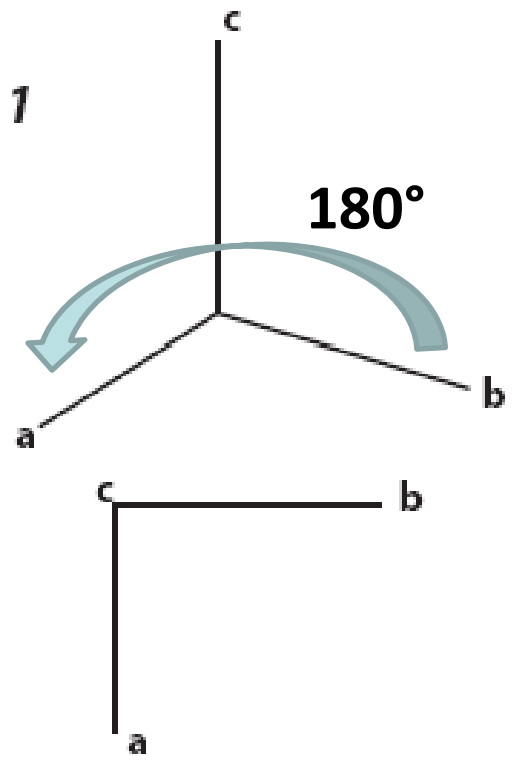
While not always immediately obvious, in most well formed crystal shapes, *axis of rotation*, *axis of rotoinversion*, *center's of symmetry*, and *mirror planes* can be recognized.

# Quantifying symmetry operations

Using linear algebra, it is possible to represent locations of points, lines or planes and operations performed on those planes by a matrix

In the mineralogical version of the standard  $x, y, z$  Cartesian system  $(a, b, c)$  a  $3 \times 3$  matrix will represent all the necessary points for any location

This method allows for a semi-quantitative approach to symmetry operations using the same terms as with Miller Indices

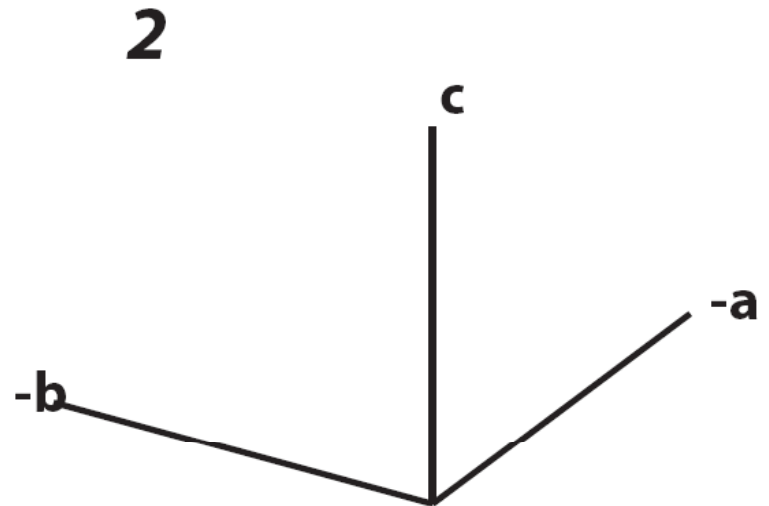
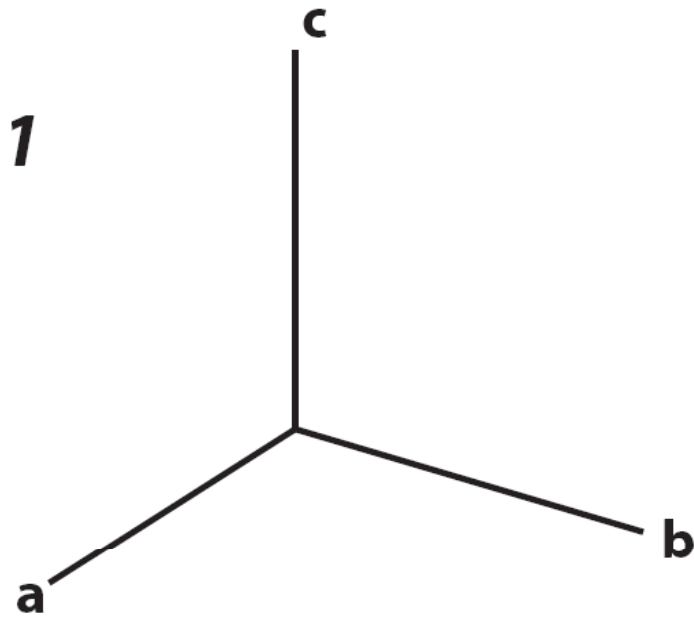


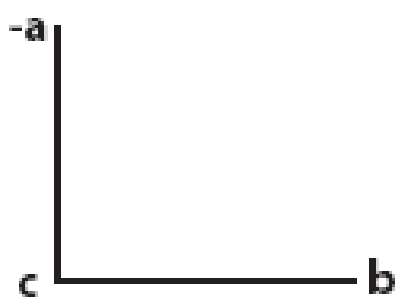
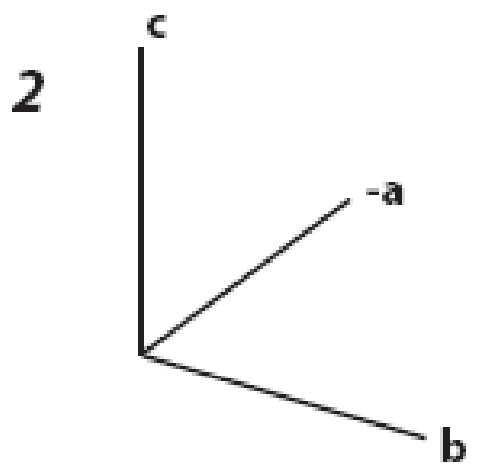
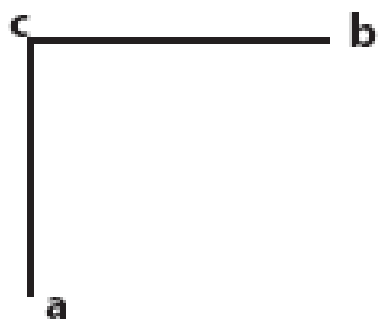
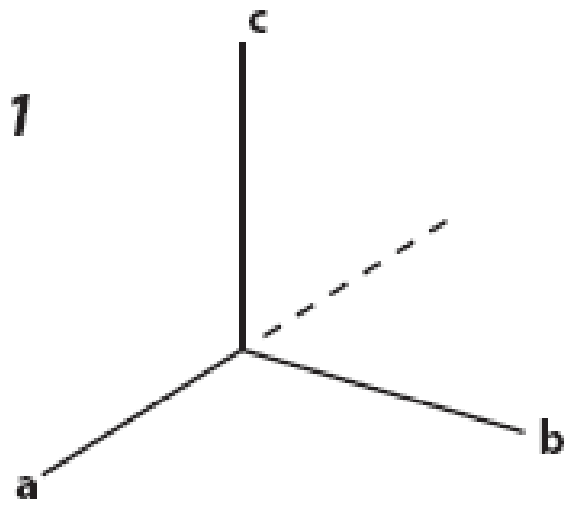
[001]2

[100]2  
Plot  
stereo

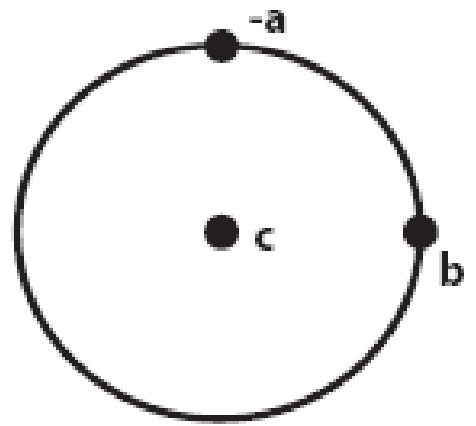
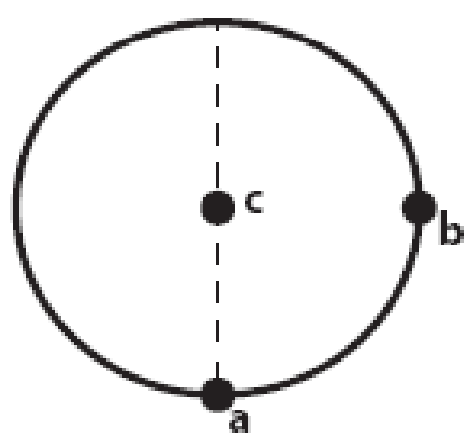
$[001]_2$

-1	0	0	a
0	-1	0	b
0	0	1	c



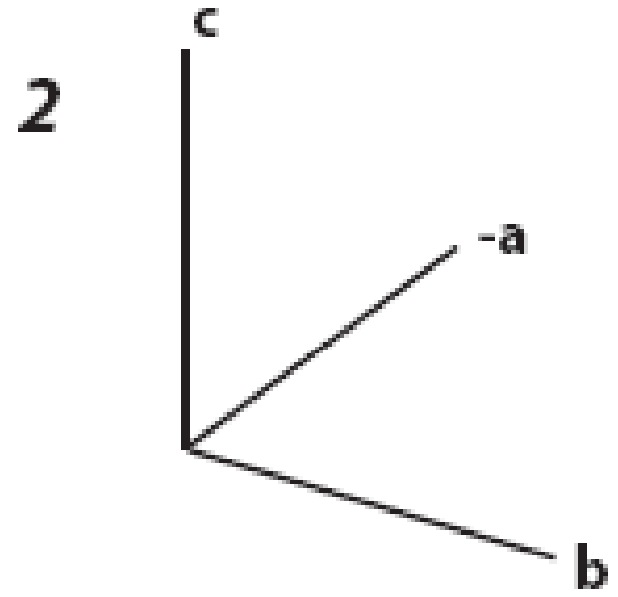
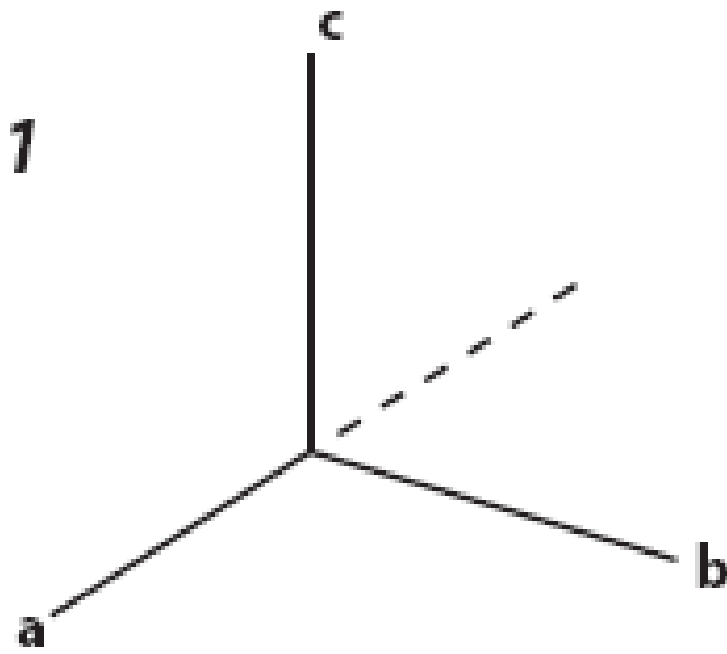


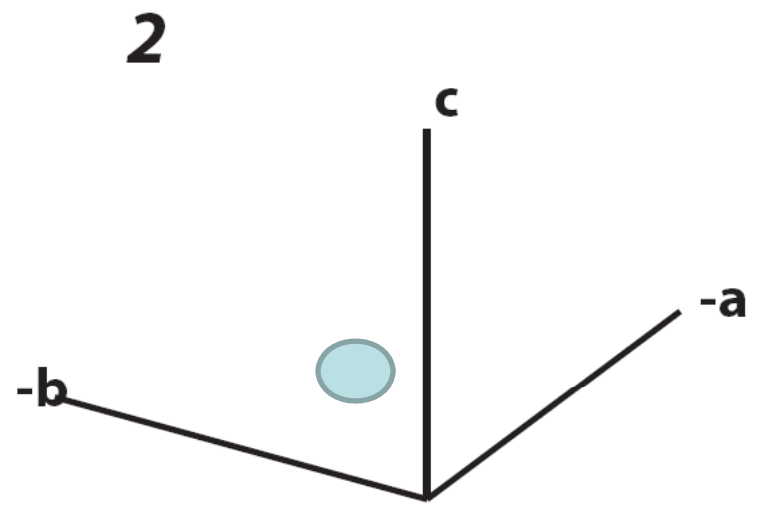
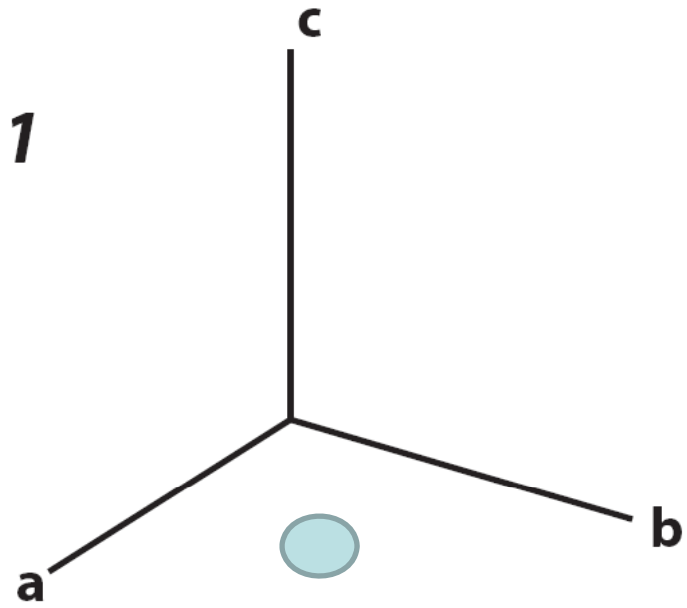
$[100]_m$



$[100]_m$

-1	0	0	a
0	1	0	b
0	0	1	c





$[100]_2$

-1	0	0
0	-1	0
0	0	1

X

1
1
0

=

-1
-1
0

$[100]_2$

Plot  
stereo

## Back to Symmetry...

All discussed operations may be combined, but the number of (i.e. unique) combinations is limited, to 32. Each of these is known as a point group, or crystal class.

The use of “point” derives from the observation that throughout each of the operations at least one point in the pattern is unmoved. Group, comes from mathematical *group theory*.

The crystal classes may be sub-divided into one of 6 crystal systems.

Each of the 32 crystal classes is unique to one of the 6(7) crystal systems:

Triclinic, monoclinic, orthorhombic, tetragonal, hexagonal and isometric (cubic)

The 7th system, trigonal, is often thought of as a sub-set of the hexagonal system.

Interestingly, while all mirror planes and poles of rotation must intersect at one point, this point itself may not be a centre of symmetry (*i*).

Of the 32 classes, 21 are without, and 11 with one.

# Crystallographic Axes

The identification of specific symmetry operations enables one to orientate a crystal according to an imaginary set of reference lines known as the *crystallographic axes*.

These are distinct and different from the classic Cartesian Axes,  $x$ ,  $y$  and  $z$ , used in other common day usage, such as plotting graphs.

With the exception of the hexagonal system, the axes are designated ***a***, ***b***, and ***c***.

The ends of each axes are designated **+** or **-**. This is important for the derivation of Miller Indices.

The angles between the positive ends of the axes are designated  **$\alpha$** ,  **$\beta$** , and  **$\gamma$** .

**$\alpha$**  lies between ***b*** and ***c***.

**$\beta$**  lies between ***a*** and ***c***.

**$\gamma$**  lies between ***a*** and ***b***.

Each of the 6 crystals systems has a unique set of crystallographic axes

**Triclinic:** Three unequal axes with oblique angles.

**Monoclinic:** Three unequal axes, two are inclined to one another, the third is perpendicular.

**Orthorhombic:** Three mutually perpendicular axes of different lengths.

**Tetragonal:** Three mutually perpendicular axes, two are equal, the third (vertical) is shorter.

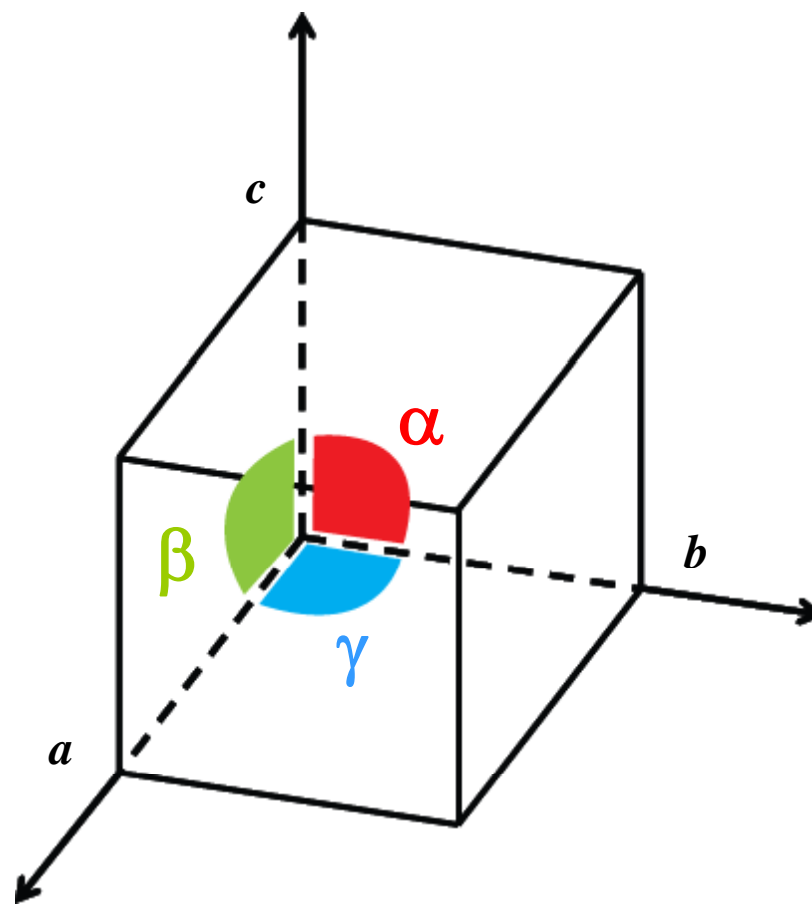
**Hexagonal:** Three equal horizontal axes ( $a_1, a_2, a_3$ ) and a 4<sup>th</sup> perpendicular vertical axis of different length.

**Cubic:** Three perpendicular axes of equal length.

**Trigonal:** Three equal axes and three equal angles that are not  $90^\circ$ .

## Triclinic: Three unequal axes with oblique angles.

- To orientate a triclinic crystal the most pronounced zone should be vertical.
- $a$  and  $b$  are determined by the intersections of  $(010)$  and  $(100)$  with  $(001)$ .
- The  $b$  axis should be longer than the  $a$  axis.



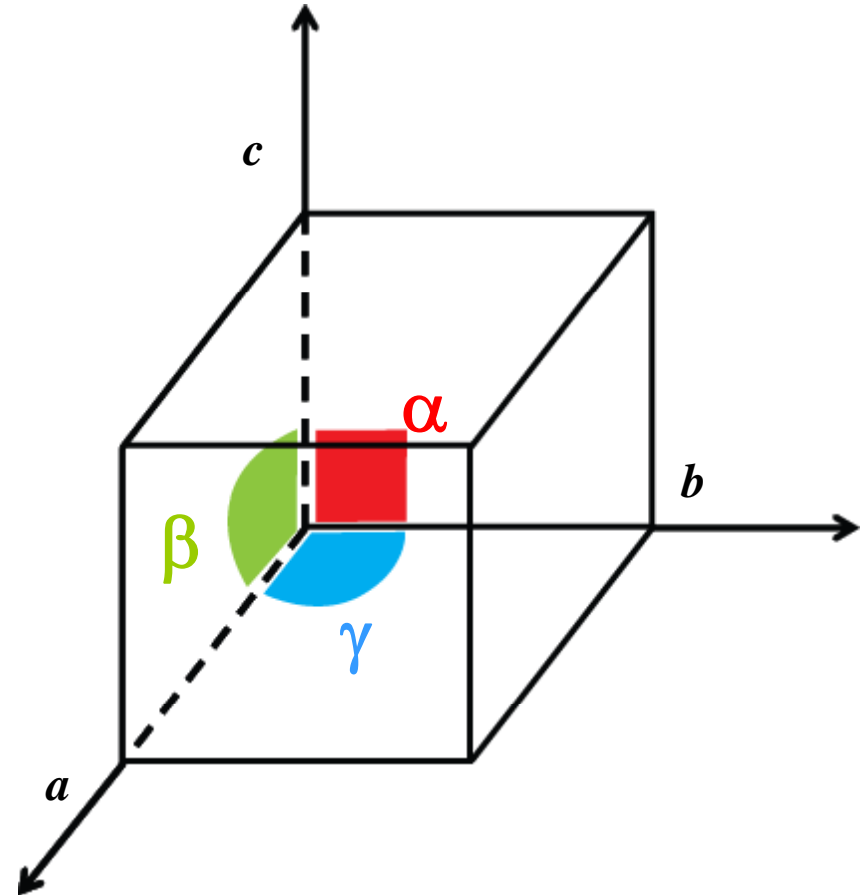
The unique symmetry operation in a triclinic system is a onefold axis of rotoinversion (i.e. equivalent to a centre of symmetry or inversion, *i*).

All forms are pinacoids – therefore must consist of two identical and parallel faces.

Common triclinic rock-forming minerals include microcline, some plagioclases, and wollastonite.

## Monoclinic: Three unequal axes, two are inclined with oblique angles, the third is perpendicular.

- Orientation of a crystal has few constraints –  $b$  is the only axis fixed by symmetry.
- $c$  is typically chosen on the basis of habit and cleavage.
- $\alpha$  and  $\gamma = 90^\circ$ .
- There are some very rare cases where  $b$  equals  $90^\circ$  giving a pseudoorthorhombic form.



The unique symmetry operation in a monoclinic system is  $2/m$  – a twofold axis of rotation with a mirror plane.

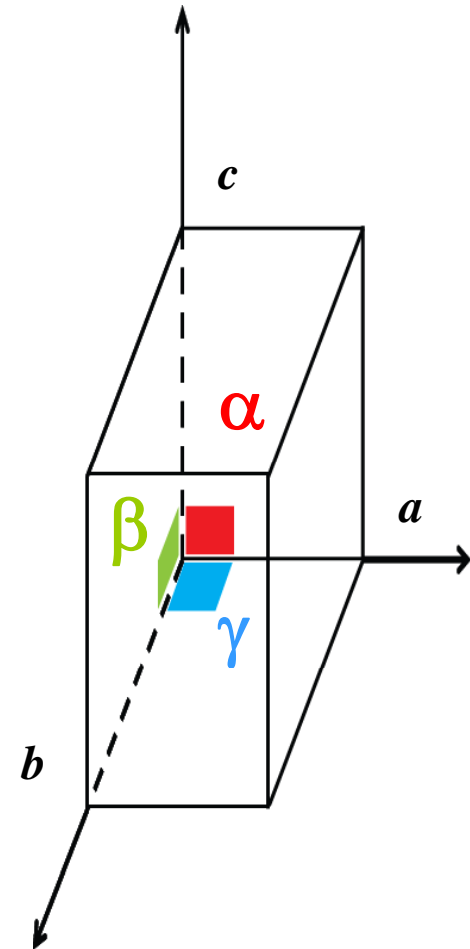
$b$  is the rotation, while  $a$  and  $c$  lie in the mirror plane.

Monoclinic crystals have two forms: pinacoids and prisms.

Common monoclinic rock-forming minerals include clinopyroxene, mica, orthoclase and titanite.

# Orthorhombic: Three mutually perpendicular axes of different lengths.

- Convention has it that a crystal is oriented such that  $c > b > a$ .
- Crystals are oriented so that  $c$  is parallel to crystal elongation.
- In this case the length of the  $b$  axis is taken as unity and ratios are calculated thereafter.



The unique symmetry operation in an orthorhombic system is  $2/m\ 2/m\ 2/m$  – Three twofold axis of rotation coinciding with the three crystallographic axes.

Perpendicular to each of the axes is a mirror plane.

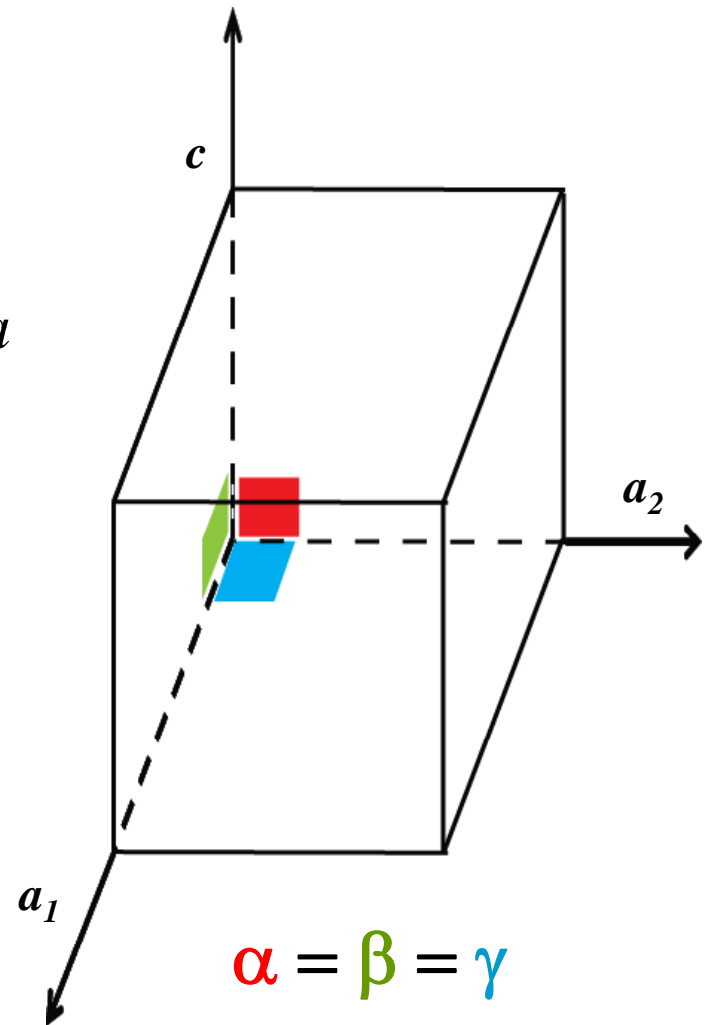
The general class for the orthorhombic system are *rhombic dipyramid*  $\{hkl\}$ .

There are three types of form in the class: pinacoids, prisms, and dipyramids.

Common orthorhombic rock-forming minerals include andalusite and sillimanite, orthopyroxene, olivine and topaz.

**Tetragonal: Three mutually perpendicular axes, two are equal, the third (vertical) is shorter.**

- The two horizontal axis in a tetragonal mineral are oriented in the plane of the horizontal. Therefore, if  $a = b$ ,  $c$  must be in the vertical.
- There is no rule as to whether  $c$  is greater or less than  $a$ .



The unique symmetry operation in a tetragonal system is  $4/m\ 2/m\ 2/m$  – The vertical axis ( $c$ ) is always a fourfold axis of rotation.

There are 4 two-fold axis of rotation: 2 parallel to the crystallographic axes  $a$  and  $b$ , the others at  $45^\circ$ .

There are 5 mirror planes.

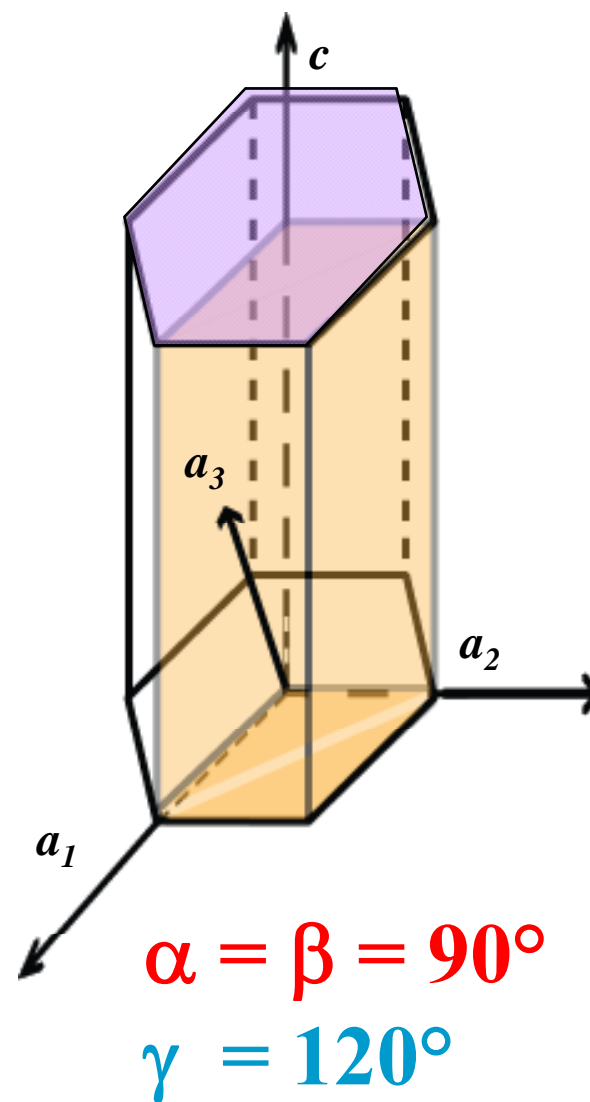
The general class for the orthorhombic system is known as the *ditetragonal-dipyramidal class*.

There are four types of form in the class: basal pinacoids, tetragonal prisms, tetragonal dipyramids, and ditetragonal prisms.

Common tetragonal rock-forming minerals include zircon, rutile and anatase, and apophyllite.

## Hexagonal: Three equal horizontal axes ( $a_1, a_2, a_3$ ) and a 4<sup>th</sup> perpendicular vertical axis of different length.

- The three horizontal axis of a hexagonal mineral are oriented in the plane of the horizontal, with  $c$  in the vertical.
- Unlike the other systems the Bravais-Miller nomenclature for crystal faces is given by 4 numbers (i.e.  $\{0001\}$ )
- The first three numbers are listed in order of  $a_1, a_2, a_3$ .



The unique symmetry operation in the hexagonal system is a six-fold axis of rotation, and the most common space group is  $6/m\ 2/m\ 2/m$ .

The vertical axis is the six-fold rotational operation, while there are a further 6 two-fold axes of rotation in the horizontal plane (3 coincide with the  $a_n$  axes).

There are 7 mirror planes.

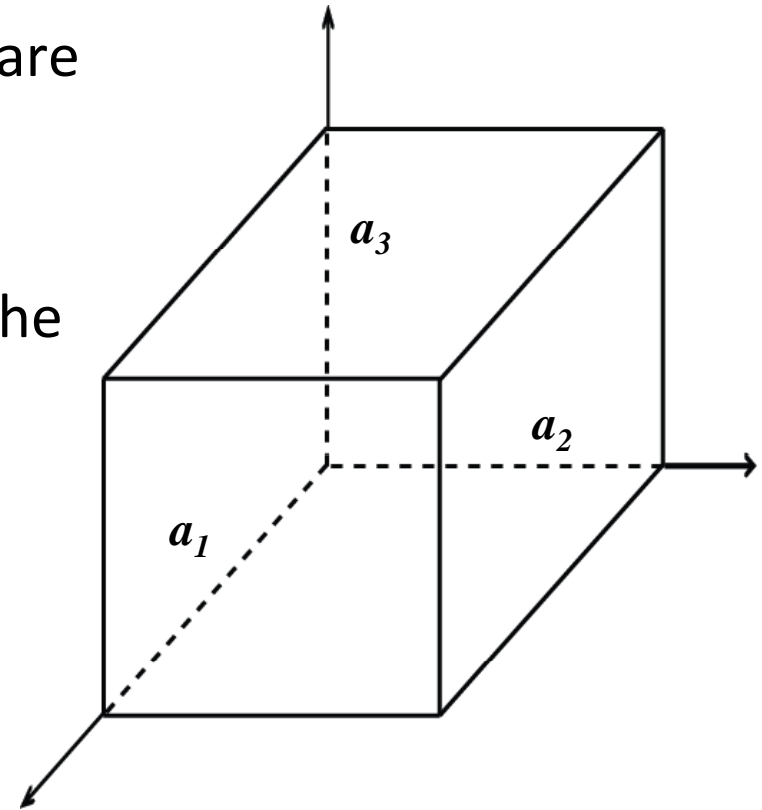
The general class for the orthorhombic system is known as the *dihexagonal-dipyramidal class*.

There are five types of form in the class: pinacoids, hexagonal prisms, hexagonal dipyramids, dihexagonal prisms, and dihexagonal dipyramids.

Common hexagonal minerals include beryl and apatite.

**Isometric (cubic): Three equal length axes that intersecting at right-angles to one another.**

- The axes are indistinguishable, as are the intersecting angles. As such all are interchangeable.
- There are 15 isometric forms, but the most common are:
  - Cube
  - Octahedron
  - Dodecahedron
  - Tetrahedron
  - Trapezohedron
  - Trisoctahedron
  - Hexoctahedron



$$\alpha = \beta = \gamma = 90^\circ$$